

System Macromodel of Agricultural Building with Aim to Energy Consumption Minimization

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Abstract

Economizing on energy intended for heating of agricultural buildings is correlative with their convenient construction from the viewpoint of both thermal and mechanical properties. Therefore, knowledge of temperature time characteristics enables building construction optimizing. This paper deals with process of recognizing elements and parameters of some particular building object, assembling its system macromodel, and analysing temperature time characteristics on its base using appropriate mathematic tools (Laplace and Fourier transformation, matrix characterizing of model parameters) and special software (ANATH). Finally, the resulting temperature time characteristics can be used for an optimal design of some planned agricultural object or for reconstruction of some existing one.

Keywords

Economy of agricultural engineering, heating energy economy, system analysis, temperature time characteristics, expenses optimizing, ambient temperature well-being.

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Introduction

Nowadays, significance of heating energy economy constantly increases. On that account, it is need to lay enhanced emphasis on building constructions design in consideration of energy consumption minimization. To process the most heat-economic building project possible, system designers must carry out a careful structural analysis from the viewpoint of active zones origin induced by floating temperature differences.

By help of a mathematic model of a building construction, it is possible to simulate and study heat behaviour of each structural element in response to time changes caused by both outdoor temperature and indoor heating system. This method is applicable for indoor non-stationary heat processes analysis as well as for computer-controlled heating optimization. (Mehta and Woods, 1981)

The idea is based on analogy between electronic and heat circuits and, therefore, the rules used for resolving electronic circuits can be analogically used for resolving heat circuits represented by the building construction mathematic model. For easy computer algorithmizing, characteristics

of the time heat responses are "harmonized" by use of the Fourier transformation (Bracewell, 1999). The model environment is described by help of the finite element method with composed parameters (thermal conductivity, thermal capacity, temperature controlled heat flow sources) and elements with spread parameters (where geometric dimensions do not allow to consider insulation resistance and thermal capacity as separate parameters) (Dong et al., 2015).

First time, this method was introduced by Professor Jiří Pánek, the former dean of the Faculty of Civil Engineering at CTU in Prague and, afterwards, it was enhanced by Professor Petr Moos and Associate professor Dalibor Vytlačil, both of them of the same institute. Later (in 1993), senior lecturer Vít Malinovský worked up a method and carried out its application on particular building constructions together with a comparative analysis of the obtained results. At the Department of Building Equipment at CTU, the team led by Professor Miroslav Jokl developed the special application ANATH for analysis and synthesis intended for calculating heat responses at different building structural configurations (Jokl, 1989).

Materials and methods

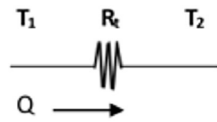
Mathematic model construction

To be able to perform matrix analysis of heat sources inside a building object, it is necessary to design an appropriate model of a building construction. The first step includes using some applicable tools from the field of system projecting determined for identification an existing or planned system within the real world and, consequently, transforming the identified system into a mathematic model form (Sangi et al., 2016). This form should take into consideration the above mentioned analogy between electronic and heat circuits for physical parameters to be definitely assigned and processed by the analytic software.

Description of model with concentrated elements

Description of model with concentrated elements deals with characterizing system properties by means of discrete (discontinuous) elements. The method proceeds from analogy between heat and electric current flow. The system consist of the following elements:

a) *Thermal resistance* is a point through which a heat flow Q passes. The Q heat flow replaces an integral representation with the Laplace image form.



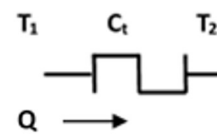
$$Q = \frac{T_1 - T_2}{R_t} \quad (1)$$

Q heat flow [W];

T_1, T_2 temperatures before and behind a point of resistance [°C];

R_t thermal resistance [W.m⁻¹.K⁻¹].

b) *Thermal capacity* is an element simulating the system accumulation properties.



$$\delta T = \frac{1}{C_t} \cdot \int q(t) \cdot dt = \frac{1}{C_t \cdot p} \cdot Q \quad (2)$$

Q heat flow [W];

T_1, T_2 temperatures before and behind a point of resistance [°C];

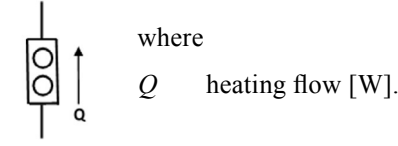
δT difference of temperatures after heat passing through an element [°C];

C_t thermal capacity [J.m⁻¹.K⁻¹];

p Laplace operator [-];

t, dt time and time increase [s],

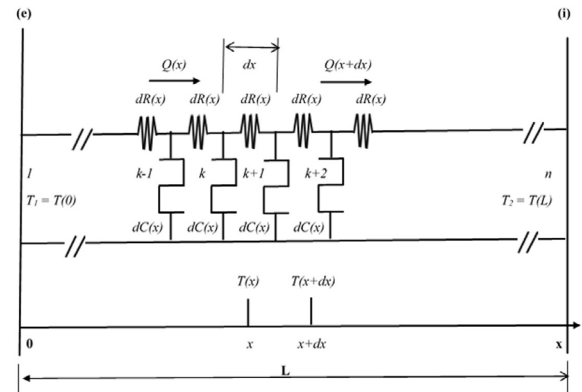
c) *Heat flow resource* is an element representing a regulated or unregulated heating system.



A heat flow resource can be regulated or unregulated. The regulated resource is characterized by the regulation factor g determining a change of temperature by 1 °K in a point of a sensor (controlling temperature point).

Description of model with spread elements

Description of model with spread elements solves inaccuracies of the previous description caused by assumption that only condition of all materials predominates (i.e. polystyrene – thermal resistance, concrete and stone – thermal capacity, etc.). However, other conditions of the most of materials cannot be omitted because both heat resistance and thermal capacity are functions of the element's thickness and, that is why, it is not possible to consider each condition separately and compile the result afterwards (Moziraji et al., 2014).



Source: own processing

Figure 1: Scheme of model with spread elements.

Q heat flow [W];

(e), (i) exterior, interior;

x variable of thickness ($x = 0$ outer surface; $x = L$ inner surface) [m];

L thickness of the whole perimeter construction [m];

$dR(x)$ increase of thermal resistance within the distance element dx [W.m⁻¹.K⁻¹];

$dC(x)$ increase of thermal capacity within the distance element dx [J.m⁻¹.K⁻¹];

$T(x)$ temperature in the point x [°C];

$Q(x)$ heating flow in the point x [W].

Mathematic definition of model with spread elements

The relations between quantities characterizing the model with spread elements is based on the following formulas:

$$\frac{\sigma T(x, t)}{\sigma x} = R(x) \cdot Q(x, t) \quad (3)$$

and

$$\frac{\sigma Q(x, t)}{\sigma x} = C(x) \cdot \frac{\sigma T(x, t)}{\sigma t} \quad (4)$$

where

- T temperature [$^{\circ}\text{C}$];
- Q heating flow [W];
- R thermal resistance [$\text{W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$];
- C thermal capacity [$\text{J} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$];
- x position of examined point [m];
- t time [s].

The temperature equation results from deriving the formula (3) and substituting the result into the formula (4).

$$\frac{\sigma^2 T(x, t)}{\sigma^2 x} - \frac{1}{R(x)} \cdot \frac{\sigma R(x)}{\sigma x} \cdot \frac{\sigma T(x, t)}{\sigma x} - R(x) \cdot C(x) \cdot \frac{\sigma T(x, t)}{\sigma t} = 0 \quad (5)$$

The heating flow results again from derivate the formula (3) and substituting the result into the formula (5).

$$\frac{\sigma^2 Q(x, t)}{\sigma^2 x} - \frac{1}{R(x)} \cdot \frac{\sigma R(x)}{\sigma x} \cdot \frac{\sigma Q(x, t)}{\sigma x} - R(x) \cdot C(x) \cdot \frac{\sigma Q(x, t)}{\sigma t} = 0 \quad (6)$$

The formulas (5) and (6) create a matrix differential equation with three variables. The time equation is transformed by means of Laplace transformation to the following form:

$$T(x, p) = A_1 \cdot e^{-\tau x} + A_2 \cdot e^{-\tau x} \quad (7)$$

and

$$Q(x, p) = B_1 \cdot e^{-\tau x} + B_2 \cdot e^{-\tau x} \quad (8)$$

where

- $\tau^2 = p \cdot C_i \cdot R_i$ heating constant;
 - R_i specific thermal resistance [$\text{W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$];
 - C_i specific thermal capacity [$\text{J} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$];
- and contains two linearly dependent constants:

$$B_1 = \tau_0 \cdot A_1 \quad (9)$$

and

$$B_2 = \tau_0 \cdot A_2 \quad (10)$$

where the specific heating constant is of the following form:

$$\tau_0 = \sqrt{\frac{R_t}{p \cdot C_t}} \quad (11)$$

For the model, the boundary conditions are determined: temperature T_2 on the inner surface ($x = L$) and heating flow Q_2 , and temperature T_1 on the outer surface ($x = 0$) and heating flow Q_1 . After substitution of (9), (10), and (11) to the equations (07) and (08) and examining the boundary conditions, the following matrix form arises (Draghici et al., 1998):

$$\begin{bmatrix} T_1(p) \\ Q_1(p) \end{bmatrix} = \begin{bmatrix} \cosh \tau d & \tau_0 \sinh \tau d \\ \frac{1}{\tau_0} \sinh \tau d & \cosh \tau d \end{bmatrix} \cdot \begin{bmatrix} T_2(p) \\ Q_2(p) \end{bmatrix} \quad (12)$$

The middle matrix $[A]$ contains all information on heating change of a layer with L , C , and R parameters. For multilayer constructions, the resulting matrix $[A]$ can be obtained by multiplying particular matrixes of individual construction layers from the outer surface to the inner one. For a construction consisting of n layers, the aggregate matrix is of the form (Pöttgen et al., 2016):

$$[A] = [A_1] \cdot [A_2] \cdot [A_2] \cdot \dots \cdot [A_n] \quad (13)$$

For computer processing, performing Fourier transformation ($p = j \cdot \Omega$) is very useful because it can transform time courses to harmonic ones (Sonderegger, 1977).

$\tau^2 = j \cdot \Omega \cdot C_i \cdot R_i$ heating constant;

complex unit j [-]; angular frequency

Ω [s^{-1}];

specific thermal resistance related to 1 m' R_i [$\text{W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$];

specific thermal capacity related to 1 m' C_i [$\text{J} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$];

$\tau_0 = \frac{R_t}{j \cdot \Omega \cdot C_t}$ specific heating constant;

$\Omega = \frac{2\pi}{T}$ angular frequency [s^{-1}]; period T [s];

$\Theta = \Omega \cdot C_i \cdot R_i \cdot L^2$ heating flow;

thickness of spread parameter L [m];

total thermal resistance $R_i \cdot L$ [$\text{W} \cdot \text{K}^{-1}$];

total thermal capacity $C_i \cdot L$ [$\text{J} \cdot \text{K}^{-1}$]

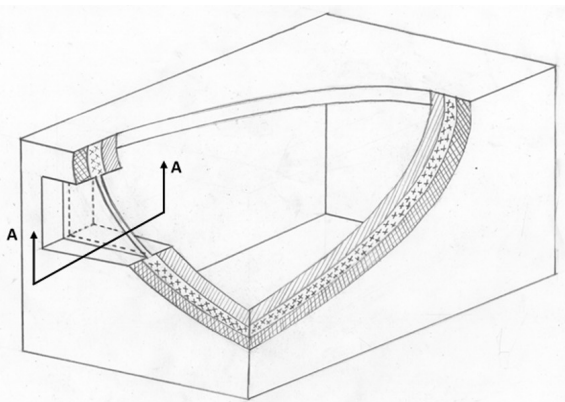
By dividing (12) according to the De Moivre's formula to a real and complex part, the coefficients of the complex matrix $[A]$ arise. The real part describes a module (maximum deviations of heating cycle) while the complex part describes a phase (time delay from exciting change) (Moos, 1989).

Analysis of barn building model

The following procedure consists of processing expected data of exterior time temperature courses by means of Fourier transformation representing independently variable quantities causing temperature changes inside the building constructions (Stupka et al., 2014). Next, important inner and outer points of around the construction – so called thermal nodes – are assigned by unique numerals. These numerals create the building construction heat model including heat relations to the monitored subjects inside the building construction (Wachowicz et al., 2016).

Barn model construction

First, for easier projection, a simplified barn building 3D-model (see Figure 2) is designed. For this purpose, a single-space building object of three-layer perimeter wall was chosen to represent an initial base for a particular scheme creation (Svoboda, 2012). The spatial projection contains marking of a typical wall cross-section A-A that is shown on the upper part of Figure 2. Also a glass-walled part is taken into the account for temperature changing calculation (Malinovský, 1993).



Source: Author's own work

Figure 2: Simplified barn building 3D-model.

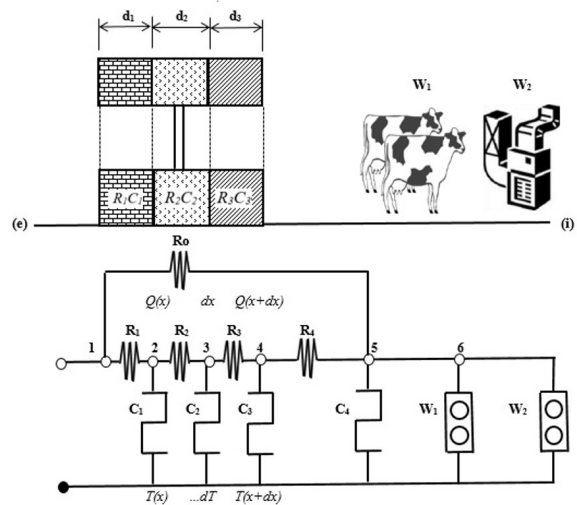
This 3D-model allows to create the particular scheme of a thermal circuit (shown on the bottom part of Fig. 2) representing a construction part of the barn building. The thermal circuit is used for entering input parameters for calculation processed by the ANATH application (Zajicek and Kic, 2014).

The ANATH application serves for calculating thermal responses of the system (object model) for changes of the outdoor temperature and, concurrently, for simulating situations when the elements and their parameters affecting the global thermal system behaviour are changing. It results from data containing recorded temperature values in the course of 24 hours. The curves representing temperature courses within the given time unit are not harmonic and are not in accordance with goniometrical functions. However, by help of Fourier transform, these non-harmonic but periodical curves can be replaced by a sum of goniometrical functions (Moos and Vytlačil, 1991):

$$T_i(\Omega \cdot t) = |M_i| \cdot e^{j \cdot \Phi_i} \quad (14)$$

where

- M_i modules of individual harmonic items Ω , 2Ω , $3\Omega \dots n\Omega$ (frequency functions);
- j complex unit;
- Φ_i argument of harmonic item



Notes:

- d_i individual construction layers' thicknesses;
- W_1 unregulated heating source – cattle (Chloupek, 1992);
- W_2 regulated heating source – heating system;
- C_i individual construction layer thermal capacities;
- R_i individual construction layer thermal resistances;
- $T(x)$ temperature as function of distance x [$^{\circ}\text{C}$];
- $Q(x)$ heating flow as function of distance x [W]

Source:

Figure 3: Barn building heating scheme.

Calculation procedure

For describing the barn building thermal system, the system matrix equation has to be created. The presence of the heat q within the barn construction is represented predominantly

in the form of the heating flow Q considered as an agent. For this quantity, the principle of agents conservation states can be expressed as following (Price and Smith, 1995):

$$\sum_{i=1}^n \Delta Q_i = 0 \quad (15)$$

where

ΔQ_i increases of individual heating flows components inside barn construction.

This equation can be expressed in the developed form:

$$[Q] - [h] \cdot [T] = 0 \quad (16)$$

where

$[Q]$ matrix of external heating flows;

$[h]$ system matrix;

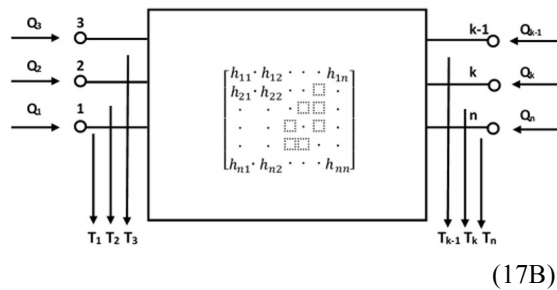
$[T]$ matrix of temperatures in heating nodes.

The matrices $[Q]$, $[h]$, and $[T]$ contain Laplace images of the corresponding time functions.

After separating the external heating flows matrix $[Q]$ to the left, the following matrix equation (in full form) origins:

$$\begin{bmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_n \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1n} \\ h_{21} & h_{22} & \cdots & \square \\ \vdots & \vdots & \square & \vdots \\ h_{n1} & h_{n2} & \cdots & h_{nn} \end{bmatrix} \cdot \begin{bmatrix} T_1 \\ T_2 \\ \vdots \\ T_n \end{bmatrix} \quad (17A)$$

This matrix can be seen as the following scheme representing the construction heating system exposed by the external heating flows $Q_1, Q_2 \dots Q_n$ in the heating nodes $T_1, T_2 \dots T_n$.



The temperature values and their time courses are the most important quantities for state changes examining in the given points and, therefore, the equation (17) must be adjusted so that the

heating nodes matrix would be in the left side:

$$[T] - [h]^{-1} \cdot [Q] \quad (18)$$

or in the full form:

$$[h]^{-1} = \frac{1}{\Delta} \begin{bmatrix} \Delta_{11} & \Delta_{21} & \cdots & \Delta_{n1} \\ \Delta_{12} & \Delta_{22} & \cdots & \Delta_{n2} \\ \vdots & \vdots & \square & \vdots \\ \Delta_{1n} & \Delta_{2n} & \cdots & \Delta_{nn} \end{bmatrix} \quad (19)$$

where

Δ determinant of $[h]$ matrix;

Δ_{ij} algebraic complements to appropriate $[h]$ matrix elements.

Supposing only exterior represents a source of the heating flow Q_1 and this environment is characterized by the T_1 temperature, another heating flows $Q_2, Q_3 \dots Q_n$ are of zero value, nevertheless, inner heating flows within the construction are time dependent values in relation to both Q_1 and T_1 changes. Further, a course of temperature changing in the k point (k heating node) is of essential importance. Dependence between temperatures T_1 and T_k is arises from the following equation system (Moos, Vytlačil 1991):

$$\begin{aligned} T_1 &= \frac{1}{\Delta} (\Delta_{11} \cdot Q_1 + \Delta_{21} \cdot Q_2 + \cdots + \Delta_{n1} \cdot Q_n) \\ T_2 &= \frac{1}{\Delta} (\Delta_{12} \cdot Q_1 + \Delta_{22} \cdot Q_2 + \cdots + \Delta_{n2} \cdot Q_n) \\ &\vdots \\ T_k &= \frac{1}{\Delta} (\Delta_{1k} \cdot Q_1 + \Delta_{2k} \cdot Q_2 + \cdots + \Delta_{nk} \cdot Q_n) \\ &\vdots \\ T_n &= \frac{1}{\Delta} (\Delta_{1n} \cdot Q_1 + \Delta_{2n} \cdot Q_2 + \cdots + \Delta_{nn} \cdot Q_n) \end{aligned} \quad (19)$$

Considering the above determined conditions, only the following two adjusted equations are used:

$$\begin{aligned} T_1 &= \frac{1}{\Delta} \Delta_{11} \cdot Q_1 \\ T_k &= \frac{1}{\Delta} \Delta_{1k} Q_1 \end{aligned} \quad (20a; 20b)$$

The impulse heating response are given as a fraction of both the equations (20a; 20b).

$$L\{f(t)\} = \frac{T_k(p)}{T_1(p)} = \frac{\Delta_{1k}}{\Delta_{11}} \quad (21)$$

The time course of the temperature $T_k(t)$ in the k point depend on external temperature course $T_j(t)$ can be determined either by means of Laplace transformation:

$$T_1(t) = \mathcal{L}^{-1} \left\{ \frac{\Delta_{1k}}{\Delta_{11}} \cdot T_1(p) \right\} \quad (22)$$

or, after substituting the Laplace operator for $p = j\omega_p$ by means of the Fourier transformation:

$$T_1(t) = \mathcal{F}^{-1} \left\{ \frac{\Delta_{1k}}{\Delta_{11}} \cdot T(j\omega_t) \right\} \quad (23)$$

as a sum of harmonic components from the heating Fourier spectre. (Lloyd et al., 1978)

If the $[h]$ system matrix is known then determining a time dependent temperature course in some selected heating nod is possible from the equations (20a; 20b). The $[h]$ matrix can be compiled with a help of the heating model. Such the model with concentrated elements is shown on Figure 3. The requested $[h]$ matrix is of the table form consisting of n rows and n columns where the n variable represents a number of the system heating nodes. These heating nodes are coupled through the thermal resistances $R_{i'}$, R_{i2} ,... and thermal capacities $C_{i'}$, C_{i2} in the model. The main diagonal elements of the $[h]$ matrix create sums of reciprocal values of thermal resistances coupled to numerically appropriate heating nodes (Evola and Marletta, 2013). The other elements are created by reciprocal values of thermal resistances with minus signs coupling individual heating nodes. Since also inertial construction properties are considered in the system due to thermal capacities, the Laplace images of heating flows and temperatures fractions are added to the reciprocal values of thermal resistances in the i^{th} nod with the attached thermal capacity (Moos, 1989):

$$\frac{Q_i(p)}{T_i(p)} = pC_{ti} \quad (24)$$

The partial $[h]$ system matrix is of the following form (25):

	1	2	3
1	$1/R_{i2}$	$-1/R_{i2}$	
2	$-1/R_{i2}$	$1/R_{i2} + 1/R_{i1} + p \cdot C_{i1}$	$-1/R_{i1}$
3		$-1/R_{i1}$	$1/R_{i1} + \dots$

(25)

Sources of the heating flow in the i^{th} nod – Q_i controlled by the temperature T_j in the j^{th} nod can be expressed by the linear equation:

$$Q_i = g \cdot T_j \quad (26)$$

where g is a regulation factor – are in the $[h]$ matrix represented so that the transfer heating conductivity g performing the regulation factor function is added to the element in the i^{th} row and j^{th} column of the $[h]$ system matrix.

Between each pair of heating nodes i and j , there is a construction element described as the system with spread parameters and, from that, a partial matrix of the second-order (its compact form in 26)

$$\begin{bmatrix} Q_i \\ Q_j \end{bmatrix} = \begin{bmatrix} h_{ii} & h_{ij} \\ h_{ji} & h_{jj} \end{bmatrix} \cdot \begin{bmatrix} T_i \\ T_j \end{bmatrix} \quad (27)$$

is derived and its partial parameters h_{ii} , h_{ij} , h_{ji} , and h_{jj} also being functions of Laplace operator p and, further, functions of the construction geometrical dimensions (wall thickness for example) are added to appropriate elements of the $[h]$ matrix.

The partial section of the construction described by the equation (27) can consist of several layers (see Fig 3). The partial layers can be suitably characterized by the following matrix equation:

$$\begin{bmatrix} T_1(p) \\ Q_1(p) \end{bmatrix} = \begin{bmatrix} \cosh \gamma d & \varrho_0 \sinh \gamma d \\ \frac{1}{\varrho_0} \sinh \gamma d & \cosh \gamma d \end{bmatrix} \cdot \begin{bmatrix} T_2(p) \\ Q_2(p) \end{bmatrix} \quad (28)$$

where

$$\gamma = \sqrt{p \cdot R_i \cdot C_i} \quad \text{heating constant;}$$

$$\varrho_0 = \sqrt{\frac{R}{pC_i}} \quad \text{specific heating constant;}$$

R_i specific thermal resistance [$\text{W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$];

C_i specific thermal capacity [$\text{J} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$].

The resulting function

$$\begin{bmatrix} \cosh \gamma d & \varrho_0 \sinh \gamma d \\ \frac{1}{\varrho_0} \sqrt{pR_iC_i} & \cosh \gamma d \end{bmatrix} \cdot \begin{bmatrix} A & B \\ C & D \end{bmatrix} \quad (29)$$

can be obtained as a product of partial matrices (28) for separate construction layers:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \cdot \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \cdot \begin{bmatrix} A_3 & B_3 \\ C_3 & D_3 \end{bmatrix} \quad (30)$$

The matrix (29) is transformed on the form compatible with the system matrix according to the following transformation function:

$$[h]_{ij} = \frac{1}{B} \cdot \begin{bmatrix} A & -1 \\ -1 & D \end{bmatrix} \quad (31)$$

The triple layer construction according to Figure 3 (lower scheme) includes the resulting thermal resistance of separate layers ΣR_i ; $i = 1, \dots, 4$ (R_i includes heating properties of heat transfer from exterior and R_4 heating transfer from the wall to interior) and thermal resistance of the window R_o . Further, the construction includes the resulting thermal capacities of separate layers ΣC_i ; $i = 1, \dots, 4$ (C_4 represents thermal capacity of interior). The unregulated heating source heating system W_1 (cattle) supplies the interior by the heating flow Q_5 and the regulated heating source W_2 (heating system regulated by the temperature T_4 measured on the inner surface of the perimeter wall; Q_6 is equal to regulation factor g) supplies the interior by the heating flow Q_6 . According to the described procedure, the following system matrix $[h]$ can be compiled (equation 32, this page below) (Moos and Vytlačil 1991):

where:

- k sum of heat transfer coefficients Σk_i ; $i = 1, 2, 3$ of all layers;
- k_0 heat transfer coefficients of window.

The Laplace image of impulse characteristic is expressed as the fraction of the algebraic complements Δ_{11} and Δ_{15} (question 33, this page below):

If the temperature T_1 suddenly decreases in the time $t = t_0$ by x °C the change can be expressed by means of the Laplace image:

$$T_5(p) = \frac{\Delta_{15}}{\Delta_{11}} \cdot \frac{X}{p} \quad (34)$$

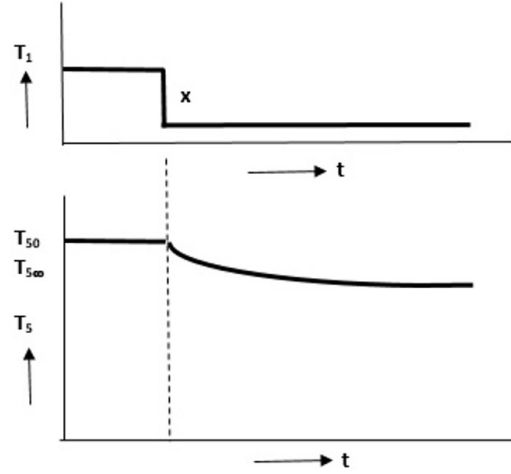
$$[h] = \begin{bmatrix} k + k_0 & -k_1 & -k_2 & -k_3 & -k_0 \\ -k_1 & k + 1/R_1 + p \cdot C_1 & -1/R_2 & -1/R_3 & -1/R_4 \\ -k_2 & -1/R_1 & k + 1/R_2 + p \cdot C_2 & -1/R_3 & -1/R_4 \\ -k_3 & -1/R_1 & -1/R_2 & k + 1/R_3 + p \cdot C_3 & g - 1/R_4 \\ -k_0 & -1/R_1 & -1/R_2 & g - 1/R_3 & k_0 + 1/R_4 + p \cdot C_4 \end{bmatrix} \quad (32)$$

$$\frac{T_5}{T_1} = \frac{\Delta_{15}}{\Delta_{11}} = \frac{\frac{k_0}{C_4} \cdot \left[\frac{k}{k_0 C_1} \cdot (1/R_4 - g) + \frac{k + 1/R_4}{C_4} + p \right]}{p^2 + p + \left(\frac{k_0 + 1/R_4}{C_4} + \frac{k_0 + 1/R_4}{C_1} \right) + \frac{1/R_4 \cdot g + k \cdot k_0 + 1/R_4 \cdot k + 1/R_4 \cdot k_0}{C_1 \cdot C_4}} \quad (33)$$

so that the temperature course T_5 within the interior can be computed by the help of the reversal Laplace transformation of the following equation:

$$T_1(t) = \mathcal{L}^{-1} \left\{ \frac{\Delta_{15}}{\Delta_{11}} \cdot \frac{X}{p} \right\} \quad (35)$$

The course of the temperature response is shown in the Figure 4.



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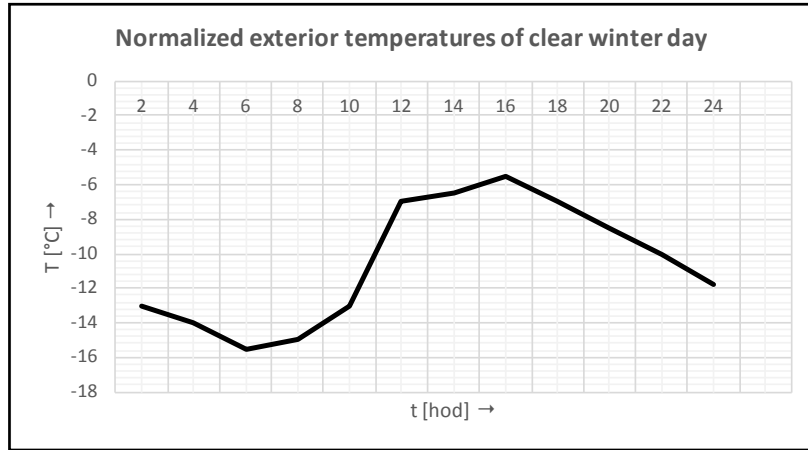
Figure 4: Time characteristics of temperature changes.

Results and discussion

Analysis of computed outputs

For analysis, the barn whose construction heating scheme is shown on Fig. 3 was used. As input data, the following set of input information was entered (Veverka et al., 1992):

- exterior temperature values was adopted from the temperature standard for a clear winter day (Figure 5).
- thermal resistances of individual materials R_i [$\text{W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$] (see Table 1);
- thermal capacities of individual materials C_i [$\text{J} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$] (see Table 1);



Source:

Figure 5: Normalized exterior temperatures of clear winter day.

Material	Thickness [m]	Thermal resistance R_i [W.m ⁻¹ .K ⁻¹]	Thermal capacity C_i [J.m ⁻¹ .K ⁻¹]
A) Reinforced concrete cladding			
Reinforced concrete	0.225	0.059	840
Polystyrene	0.145	2.5	1550
Heraklit	0.115	0.357	1590
B) Ceramic cladding			
CD-IVA-C	0.14	0.238	690
Polystyrene	0.04	2.5	1550
CD-IVA-C	0.14	0.238	690
C) Plasterboard cladding (no supporting structure; used with any other construction)			
Plasterboard	0.03	0.625	1380
Prefizol	0.10	2.0	920
Eternit boards	0.02	0.2	960

Source: Rochla (1983)

Table 1: Cladding configurations.

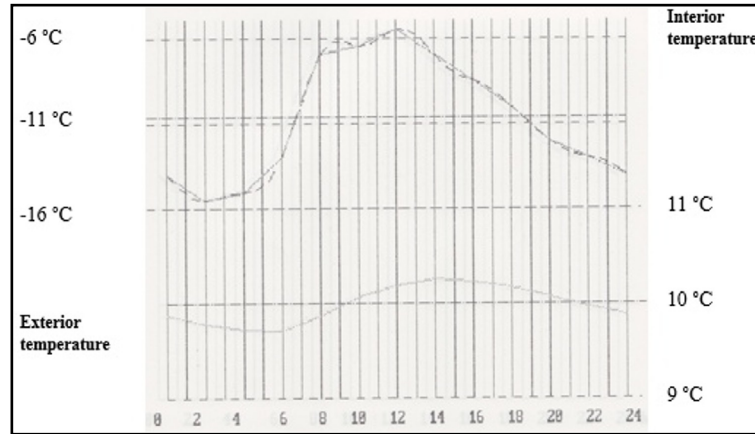
- thermal resistances of window R_i [0.04 W.m⁻¹.K⁻¹];
- unregulated heating source W_1 [cattle; constant 125 W.K⁻¹];
- regulated heating source W_2 [heating system; constant 100 W.K⁻¹];

Processing the input data using ANATH application is carried out in several steps:

- 1) Fourier transformation of temperature time course being an independent variable causing temperature changes within the construction.
- 2) Assigning numbers to individual points – or heating nodes – within and out of the construction whose temperature should be examined.
- 3) Creating a heating model of the building construction and its couplings to the examined object within the building's interior.
- 4) Forming a matrix of heat penetration and heating absorbability through given parts of the construction. Completing the elements representing the heating system and adding the submatrices characterizing the system with spread parameters. The result is the system matrix.
- 5) Obtaining the image of the system's heat impulse responses in the nodes important from the viewpoint of heating gradients by the help of quotients of the system matrix algebraic complements and optimizing the function of the heating system.
- 6) Determining the temperature course in the examined points by reverse transformation of the Fourier convolution image of exterior temperature time course with the system impulse response (Moos and Vytlačil 1991).

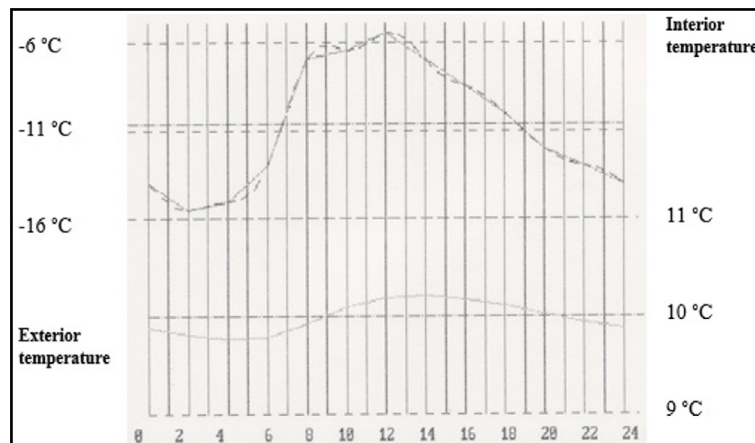
The resulting graphs consist of two parts. The first ones (situated in upper halves) represent rough (continuous lines) curves (according to Figures 6-8) and smooth interpolated (dashed lines) curves

of exterior temperatures. The second ones (situated in lower halves) represent responses of interior temperature on the exterior temperatures shown above.



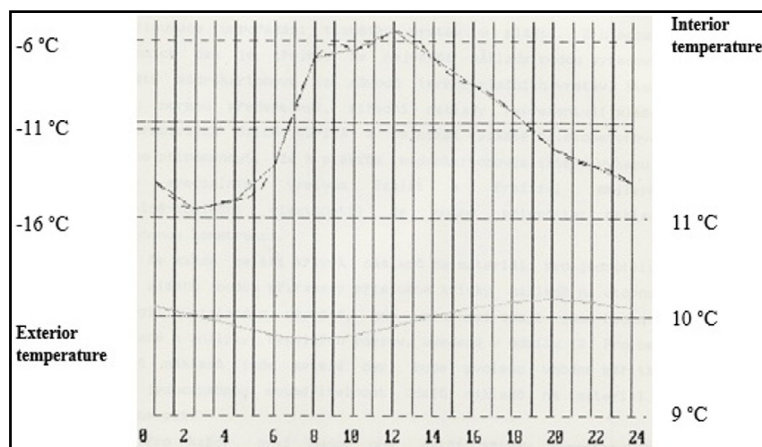
Source: authors' own processing in ANATH

Figure 6: Heating responses of Cladding A.



Source: authors' own processing in ANATH

Figure 6: Heating responses of Cladding B.



Source: authors' own processing in ANATH

Figure 6: Heating responses of Cladding C.

Conclusion

Knowledge of time temperature characteristics enables expecting values of mechanical tension inside constructions given by thermal dilatations of structural parts in the course of non-stationary thermal processes and, afterwards, building construction optimizing. The complex analysis of temperature time characteristics can be applied for examining thermal bridges within interiors of building objects with humid ambient and with light cladding (Cooke 1975). The results provided in the form of graphs and temperature data files by the ANATH application can be used by various manners. For example, they can serve for simulating inner temperatures of agricultural buildings (cow barn, in our case) and following setting the appropriate heating plan

via programming heat regulation systems – provided the existing building objects or for designing optimal construction configuration (system of cladding layers arrangement) in case of planned objects (Hoffman and Feldman, 1981). A specific task was solved by Malinovský in 1993, when a cost curve (material costs versus heating costs) had been built on the basis of analysing heating responses at different cladding material configurations an standardized exterior temperatures. Of course, analysing thermal behaviour of building object interiors on the basis of multi-factors (temperature responses, cladding material configuration, heating system configuration, construction type, ventilation system and mode, etc.) can be used for lot of different purposes in the field agriculture engineering.

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