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On the Comparison of Akaike Information Criterion and Consistent Akaike Information Criterion in Selection of an Asymmetric Price Relationship: Bootstrap Simulation Results

H. D. Acquah

Department of Agricultural Economics and Extension, University of Cape Coast, Ghana

Abstract

Akaike's Information Criteria provide a basis for choosing between competing approaches to testing for price asymmetry. However, very little research has been undertaken to understand its performance in the price transmission modelling context. In addressing this issue, this paper introduces and applies parametric bootstrap techniques to evaluate the ability of Akaike Information Criteria (AIC) and Consistent Akaike Information Criteria (CAIC) in distinguishing between competing asymmetric price transmission models under various error and sample size conditions. Bootstrap simulation results suggest that the performance of the model selection methods depends on sample size and stochastic variance. The Bootstrap simulations further indicate that CAIC is consistent and performs better than the AIC in large bootstrap samples. The ability of the model selection methods to identify the true asymmetric price relationship decreases with increase in stochastic variance. The research findings demonstrate the usefulness of Bootstrap algorithms in price transmission model comparison and selection.

Key words

Asymmetry, Akaike's Information Criteria (AIC), Consistent Akaike Information Criteria (AIC), model selection, Bootstrapping.

Introduction

Researchers have developed alternative methods to detect asymmetric price transmission processes in agricultural markets. However there is the need to choose the best model from a set of competing models (or theories) since the different methods leads to differences in inferences and conclusions.

Information theoretic selection criteria have been developed to objectively accomplish the goal of selecting the best model from a set of competing models or theories. For instance, in signal processing problems, Seghouane and Lathauwer (2003) applied bootstrap simulations to investigate the performance of Akaike and Kullback information criteria in evaluating the number of signals. Their studies revealed that the model selection methods performed well in small samples but performance did not improve substantially in larger samples. Traditional information-theoretic criteria such as Akaike's Information Criteria (AIC) (Akaike, 1973) and lesser-known criteria such as Consistent Akaike Information Criteria (CAIC) are used for the purpose of identifying the correct asymmetric model. However, little is known about their relative performance of AIC and its extension in the asymmetric price transmission modelling context. Acquah (2010) sheds light on the relative performance of AIC and CAIC in a Monte Carlo Experimentation but did not consider the use of bootstrap techniques to analyse the relative performance of AIC and CAIC. However, little is understood about their relative performance in selecting the correct asymmetric model in bootstrap samples.

An important question which remains unanswered is how well AIC and CAIC will perform when bootstrap samples are used in the price transmission analysis. In the presence of bootstrap samples, will AIC and CAIC point to the correct model as noted in previous Monte Carlo studies? Using bootstrap methods to construct a series of new samples which are based on original data gives an advantage over the previous Monte Carlo model selection studies which makes implicit assumption about the true values of the parameters.

In order to address these issues, this paper evaluates the ability of AIC and its extension, CAIC to choose between alternative methods of testing for asymmetry in the presence of bootstrap samples. Fundamentally, the study is intended to understand the behaviour of the model selection criteria in the presence of bootstrap samples. In effect this study compares the relative performance of the well known Akaike Information Criteria with a lesser-known criterion, CAIC (Bozdogan, 1987) in terms of their ability to recover the true data generating process (DGP) in the presence of bootstrap samples. The true asymmetric data generating process is known in all experiments and the Bootstrap simulations are necessary in deriving the model recovery rates of the correct model.

The rest of the paper proceeds as follows. In the following section, an introduction of the model selection criteria is presented. This is followed by an introduction of bootstrap methods and brief description of asymmetric price transmission models. A practical application in which the performance of the model selection methods in selecting the correct asymmetric model are evaluated using Bootstrap samples is presented. Finally, the study ends with conclusions.

Materials and Methods

Model selection criteria

In order to determine the correct underlying model of a data set, one may simply suggest the most appropriate model is the one which provides the best fits to the data. This idea, however, does not work because it will always favour the most complex model among the set of competing models.

The reason is that the most complex model has more degrees of freedom and can therefore fit the data better than any other model in that set of competing models. Thus, to choose the correct model, one needs to establish a tradeoff between how well a model fits the data, which is often measured by the sum of squared residuals and the complexity of that model. In practice, higher order models have to be penalized so that the selected model would be chosen based on its suitability rather than its fidelity to data. In effect, the fundamental difference between all the existing model selection criteria is in the way by which they penalize the higher order models.

Akaike Information Criteria (AIC)

Akaike Information Criteria (Akaike, 1973) is one of the first model selection methods introduced. AIC is based on the idea that a chosen model is correct if it can sufficiently describe any future data with the same distribution and therefore AIC can be regarded as a hypothetical cross validation method. It selects a model that minimizes the expected error of the new observation with the same distribution as the data used for fitting. In short, AIC was developed to estimate the expected Kullback-Leibler distance between the true model and the estimated model. It is defined as:

$$AIC = -2 \log (L) + 2p \qquad (1)$$

Where L refers to the likelihood under the fitted model and is the number of parameters in the model. The model with minimum AIC value is chosen to be the best model.

Consistent Akaike's Information Criteria (CAIC)

Bozdogan (1987) proposed a corrected version of AIC in an attempt to overcome the tendency of the AIC to overestimate the complexity of the underlying model. Bozdogan (1987) observed that Akaike Information Criteria (AIC) does not directly depend on sample size and as a result lacks certain properties of asymptotic consistency. In formulating CAIC, a correction factor based on the sample size is employed to compensate for the overestimating nature of AIC. CAIC which reflects sample size and has properties of asymptotic consistency can be defined as:

$$CAIC = - 2 \log (L) + p [(\log n) + 1] \qquad (2)$$

Where L refers to the likelihood under the fitted model, p is the number of parameters in the model and n is the sample size. AIC differs from CAIC in the second term which now takes into account sample size n. Models that minimize the Consistent Akaike Information Criteria are selected.

The formulation of CAIC shows that AIC can be fairly extended to make it consistent, even though a practical difficulty is that consistency is a weak property (Atkinson, 1980). It can be noted that CAIC is similar to the Schwarz's (1978) criterion of $p \log n$, and that the term $[p \log n + p]$ has the effect of increasing the 'penalty term.' Consequently, the minimization of CAIC leads in general to lower dimensional models than those obtained by minimizing AIC.

The bootstrap method

Bootstrap Method introduced in Efron and Tibshirani (1993) is a resampling procedure for estimating the distribution of a statistic based on independent observations. Generally, the resampling method (bootstrap) allows us to quantitfy uncertainty by calculating parameters of interest such as standard errors and confidence intervals. Resampling procedures require fewer assumptions and give accurate results.

Bootstrapping involves repeated random sampling with replacement from the original data to create new samples referred to as the bootstrap samples. Each bootstrap sample is the same size as the original random sample and can be used to calculate the statistic of interest. The distribution of the bootstrap samples is referred to as the bootstrap distribution.

Parametric Bootstrap

In a parametric bootstrap procedure, the resampling is carried out on a parametric model.

The parametric bootstrap involves estimating regression coefficients for the original data and calculating the fitted values and residuals for each observation. Selected bootstrap samples of the residuals (ε^*) and the fixed values of the explanatory variables (x) are used to obtain the bootstrap y values. Subsequently, the bootstrap y values are regressed on the fixed values to obtain the bootstrap regression coefficients and parameters of interest.

The process where the resampled residuals are added to the original regression equation to generate new bootstrap values for the outcome variable and the coefficients of the new bootstrap regression estimated using ordinary least squares technique are outlined as follows.

1. Generate ε^* by sampling with replacement from $\hat{\varepsilon}_1, \ldots, \hat{\varepsilon}_n$
2. Form $y^* = X\hat{\beta} + \varepsilon^*$
3. Compute $X\hat{\beta}^*$ from (X, y^*)

Resampling of the residuals, adding them to the fitted values and estimating the regression coefficients is repeated a larger number of times to estimate parameters of interest with the bootstrap samples. The parametric bootstrap implicitly assumes that the functional form of the regression model is correct and that the errors are identically distributed.

Asymmetric price transmission models

The data generating process is derived from Granger and Lee (1989) Error Correction Model and can be specified as follows:

$$\Delta y_t = \beta_1 \Delta x_t + \beta_2 (y - x)_{t-1} + \varepsilon_{2,t}$$

$$\varepsilon_{2,t} \sim N(0, \delta^2) \quad (3)$$

where y and x are price series at different levels of marketing chain. In this study, y and x are generated as I(1) non stationary variables that are cointegrated and there exist an equilibrium relationship between y and x which is defined by an error correction term. The long run dynamics captured by the error correction term are implicitly symmetric. In order to allow for asymmetric adjustments, the error correction term can be decomposed into positive and negative components as follows:

$$(y - x)_t^+ = \begin{cases} (y - x)_t, & if\ (y - x)_t > 0 \\ zero & otherwise \end{cases} \quad (4)$$

$$(y - x)_t^- = \begin{cases} (y - x)_t, & if\ (y - x)_t < 0 \\ zero & otherwise \end{cases} \quad (5)$$

The resulting asymmetric model is defined as

$$\Delta y_t = \beta_1 \Delta x_t + \beta_2^+ (y - x)_{t-1}^+ + \beta_2^- (y - x)_{t-1}^- + \varepsilon_{3,t}$$

$$\varepsilon_{3,t} \sim N(0, \delta^2) \quad (6)$$

Asymmetry is introduced by allowing the speed of adjustment to differ for the positive and negative components of the Error Correction Term since the long run relationship captured by the error correction term was symmetric. Symmetry in equation (6) is tested by determining whether the coefficients (β_2^+ and β_2^-) are identical (that is H_0: $\beta_2^+ = \beta_2^-$).

Cramon-Taubadel and Loy (1996) departs from Granger and Lee's model which test for asymmetries in the adjustments in the equilibrium level to propose a complex approach to asymmetry in which asymmetries specified affects the direct impact of price increases and decreases as well as adjustments to the equilibrium level.

$$\Delta y_t = \beta_1^+ \Delta x_t + \beta_1^- \Delta x_t^- + \beta_2^+ (y - x)_{t-1}^+ + \beta_2^- (y - x)_{t-1}^- + \varepsilon_{4,t}$$

$$\varepsilon_{4,t} \sim N(0, \delta^2) \quad (7)$$

Where Δx_t^+ and Δx_t^- are the positive and negative changes in and the remaining variables are defined as in equation (7).

A formal test of the asymmetry hypothesis using the above equation is: H_0: $\beta_1^+ = \beta_1^-$ and $\beta_2^+ = \beta_2^-$. In this case, a joint F-test can be used to determine symmetry or asymmetry of the price transmission process.

Alternatively, Houck (1979) departs from the von Cramon-Taubadel and Loy (1996) model specification and proposes a model in which asymmetries specified affects the direct impact of price increases and decreases and does not take into account adjustments to the equilibrium level. The Houck method can be written as follows:

$$\Delta y_t = \beta_1^{+}\Delta x_t + \beta_1^{-}\Delta x_t^{-} \qquad \varepsilon_{5,t} \sim N(0, \delta^2) \tag{8}$$

The variables in the model are defined as in equation (7). Symmetry is tested by determining whether the coefficients (β_1^{+} and β_1^{-}) are identical (that is H_0: $\beta_1^{+} = \beta_1^{-}$).

Results and discussion

A bootstrap comparison of the performance of AIC and CAIC

The performance of AIC and CAIC in recovering the true data generating process (DGP) is investigated by simulating the effect of sample size and noise levels on model selection.

In accordance with the experimental designs of Holly et al. (2003), the value of β_1 is set to 0.5 and (β_2^{+}, β_2^{-})∈(-0.25, -0.75) are considered for the coefficients of the asymmetric error correction terms in the true model. The different models are fitted to the bootstrap data and their ability to recover the true model was measured. The recovery rates were derived using 1000 bootstrap simulations. The data generation process is defined in equation (6) and the data is simulated from the standard error correction model as follows:

$$\Delta y_t = 0.5x_t - 0.25(y_t - x_t)^{+}_{t-1} - 0.75(y_t - x_t)^{-}_{t-1} + \varepsilon \tag{9}$$

The prices y and x are generated as I (1) non stationary variables that are cointegrated. The error correction terms denotes the positive and negative deviations from the equilibrium relationship between y and x. However, we attempt to evaluate the abilities of AIC and CAIC to select the appropriate asymmetry model from competing alternatives.

The relative performance of the two model selection methods are compared in terms of their success rates or ability to recover the true data generating process (DGP) across various bootstrap sample size conditions (i.e. Model Recovery Rates) as detailed in Table 1.

For the purpose of brevity, the standard asymmetric error correction model, the complex asymmetric error correction model and the Houck's model in first differences are denoted by SECM, CECM and HKD respectively.

For each model selection method, the model recovery or success rate defines the percentages of bootstrap samples in which each competing model provides a better model fit than the other competing models. The model selection methods performed reasonably well in identifying the true model, though their ability to recover the true asymmetric data generating process (DGP) increases with increase in bootstrap sample size. In small bootstrap samples (upper part of Table 1), the model selection methods recovered at most 77.5 % of the data generating process. When the bootstrap sample size was large (Lower part of Table 1), the model selection method recovered

Experiment criterion	**Model fitted**			
	Methods	**CECM (%)**	**HKD (%)**	**SECM (DGP) (%)**
n = 50 σ = 1	AIC	18.9	5.0	76.1
	CAIC	4.4	18.1	77.5
n = 150 σ = 1	AIC	20.0	0.0	80.0
	CAIC	1.9	0.2	97.9
n = 500 σ = 1	AIC	19.0	0.0	81.0
	CAIC	1.1	0.0	98.9

Note: Recovery rates based on 1000 Bootstrap replications.
AIC: Akaike Information Criteria; CAIC: Consistent Akaike Information Criteria;
CECM: Complex Error Correction Model; HKD: Houcks Model in Differences
SECM: Standard Error Correction Model

Table 1: Relative performance of the model selection methods across sample size.

at most 98.9 % of the true model. AIC performs well in small bootstrap samples, but is inconsistent and does not improve in performance in large bootstrap samples whilst CAIC in contrast is consistent and improves in performance in large bootstrap sample size. Generally, model selection performance improved as bootstrap sample sizes increased.

Recovery rates of Consistent Akaike Information Criteria strongly depended on sample size for the true data generating process (DGP). It increased from 77.5 percent to 98.9 percent when the bootstrap sample size was increased from 50 to 500. Alternatively, recovery rates of AIC increased from 76.1 percent to 81.0 percent for the true data generating process (DGP) when the bootstrap sample size was increased from 50 to 500. Although AIC performed well in the small bootstrap samples, it did not make substantial gains in recovering the true model as the bootstrap sample size increased.

The results of the current study are consistent with the Monte Carlo Simulation experimentation of Acquah (2010) which indicated that the ability of AIC to select a true model rapidly increased with sample size but at larger sample sizes it continued to exhibit a slight tendency to select complex models whiles CAIC in contrast is consistent and improves in performance as sample size increased. Generally, these results are confirmed in the bootstrap simulation results presented in Table 1.

In order to illustrate the effects of noise level on model selection, this study considers three error sizes (σ) ranging relatively from small to large and corresponding to 1.0, 2.0 and 3.0. Using 1000 bootstrap replications, data is generated from equation (9) with the different error sizes and a sample size of 150. The data fitting abilities of alternative models are compared in relation to the true model as the error in the data generating process was increased systematically.

Table 2 shows the percentage of bootstrap samples in which the correct model (i.e. SECM) was selected or recovered among competing models by the model selection criteria as the amount of noise in the data generating process was increased. The performance of the model selection algorithms analysed declined with increasing amount of noise in the true asymmetric price transmission data generating process.

Recovery rates of Consistent Akaike Information Criteria decreased from 97.9 percent to 40.0 percent when the error size was increased from 1 to 3. Similarly, recovery rates of AIC decreased from 80.0 percent to 65.2 percent for the true data generating process (DGP) when the error size was increased from 1 to 3. Except for high noise levels CAIC outperforms AIC.

Simulating the effects of sample size and stochastic variance concurrently affirms that a small error and large sample improves recovery of the true asymmetric data generating process and vice versa as illustrated in Table 3.

With a small bootstrap sample of 50 and an error size of 2.0, the true data generating process was recovered at least 32.9 percent of the time by the model selection criteria as illustrated in upper part of Table 3. On the other hand, with a relatively large sample of 150 and error size of 0.5 at least 80.0 percent of the correct model was recovered across all the model selection methods as indicated in the lower part of Table 3. The model recovery rates of the model selection methods are derived under combined conditions of a small bootstrap sample size of 50 and large error size of 2 (i.e. Unstable conditions), and a relatively large bootstrap sample size of 150 and a small error size of 0.5 (i.e. Stable conditions). Under stable conditions,

Experiment criterion	**Model fitted**			
	Methods	**CECM (%)**	**HKD (%)**	**SECM (DGP) (%)**
n = 150 σ = 3	AIC	14.1	20.7	65.2
	CAIC	0.8	59.2	40.0
n = 150 σ = 2	AIC	18.7	4.9	76.4
	CAIC	1.3	23.8	74.9
n = 150 σ = 1	AIC	20.0	0.0	80.0
	CAIC	1.9	0.2	97.9

Note: Recovery rates percentages based on 1000 Bootstrap replications.

Table 2: Relative performance of the selection methods across error size.

Experiment criterion	Model fitted			
	Methods	CECM (%)	HKD (%)	SECM (DGP) (%)
n = 50 σ = 2	AIC	11.9	34.1	54.0
	CAIC	1.8	65.3	32.9
n = 150 σ = 0,5	AIC	20.0	0.0	80.0
	CAIC	1.9	0.0	98.1

Note: Recovery rates percentages based on 1000 Bootstrap replications.

Table 3: Effects of sample size and stochastic variance on model recovery.

model selection performance or recovery rates improves in bootstrap samples.

The results of the bootstrap simulations with regards to the effect of noise levels on model selection are generally consistent with Acquah (2010) Monte Carlo simulations which suggest that the recovery rates of the true data generating process decreases with increasing noise levels in asymmetric price transmission regression models.

An important attribute of the current study is that they generally echo existing empirical work on the performance of model selection methods in other applications. First the results of the Bootstrap simulation indicate that AIC and CAIC points to the true asymmetric price transmission model. Similarly, Tan and Biswas (2012) demonstrated via bootstrap simulation that AIC clearly identified the true data generating process in cosmological modeling framework. Using bootstrap simulation to guide the selection of the true model in multiple regression analysis, Al Mrshadi (2009) finds that AIC points to the true model. Secondly, the current study found that an AIC and related measure performs better in smaller samples. This finding is consistent with empirical applications of Seghouane and Lathauwer (2003) in signal processing modeling.

Conclusions

This study investigated the ability of AIC and its analytical extension CAIC to clearly identify the correct asymmetric model out of alternative competing models in the presence of bootstrap samples. The Bootstrap simulations results indicated that the sample sizes and noise levels are important in the selection of the true asymmetric model. With larger bootstrap sample sizes or lower noise levels, the ability of the model selection methods to identify the correct asymmetric price data generating process improved. Generally, under unstable conditions such as small bootstrap sample and large noise levels CAIC performs better than AIC. These results suggest that CAIC which corrects for sample size performs better in selecting the true asymmetric price transmission model when the number of bootstrap samples is large. The Bootstrap comparison provided sheds light on the empirical performance of the Akaike's Information Criteria and the Consistent Akaike Information Criteria in choosing an asymmetric price transmission model in the presence of bootstrap samples. Bootstrap simulation results further demonstrates the usefulness of combining Bootstrap techniques with model selection methods to identify the correct asymmetric price transmission model. Future research will investigate model selection using Bayesian methods.

Corresponding author:
Henry de-Graft Acquah
Department of Agricultural Economics and Extension, University of Cape Coast, Ghana
E-mail: henrydegraftacquah@yahoo.com

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