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Abstract

While transporting some material a circular way of the transportation is usually applied. Usually due to some capacity or time constraints or other reasons it is necessary to use more routes (i.e. more vehicles, or one vehicle must go out from its home place more times). This case is called the vehicle routing problem and there exist many types of this task because of the variety of reasons causing the necessity of use more than one route.

Practically all the vehicle routing problems belong among the so-called NP-complete or NP-hard problems. This means that there exists no effective method which would succeed in finding a precise theoretical optimum for them. In such tasks, we can employ different approximation methods which provide us with solutions similar to a theoretical optimum and acceptable as an economic optimum.

In practice, however, companies seldom pay enough attention to dealing with such problems, especially if transportation is not their principal work load and if it concerns a transportation task of a medium size.

This article presents a case study of NOPEK Bakery in Vysoké Mýto. It demonstrates the effectiveness of the approximation method during the planning of the bakery products delivery to its customers. By the optimization of one of the so-called "fast deliveries", we succeeded in the reduction of the number of vehicles needed for the delivery – about 18% – which turned out necessary. Similar savings of all "fast deliveries" in the company may lead to the reduction of tenure price (tenure fixture) by 17 mil. CZK. At the same time the profit will increase by 0.6 mil. CZK and profitability will go up by 2.5%. We also managed to ensure a balanced use of the vehicles. This made it possible for the bakery to deliver the goods to its customers in deadlines that they found more convenient.

Key words

Bakery products delivery, vehicle routing problem, heuristics (approximation method).

Anotace

Při rozvozu či svozu určitého materiálu je obvykle výhodné využívat okružní způsob. Většinou je třeba z důvodu kapacitních nebo časových omezení použít k rozvozu více tras (tj. více vozidel nebo jedno vozidlo musí jet vícekrát). Takováto úloha se nazývá trasovací problém. Vzhledem k tomu, že důvody omezení vedoucí k nutnosti použití více tras mohou být různé, trasovacích problémů existuje mnoho typů.

Téměř všechny trasovací problémy patří mezi tzv. NP-úplné nebo NP-těžké problémy. To znamená, že neexistuje žádná efektivní metoda, která by dokázala najít jejich přesné teoretické optimum. Pro takové úlohy lze ale používat různé aproximační metody, které dávají řešení blízká teoretickému optimu přijatelná jako ekonomická optima.

V praxi ovšem firmy často nevěnují řešení těchto problémů příliš pozornost, zvláště pokud doprava není jejich hlavní pracovní činností a pokud se jedná o dopravní úlohy střední velikosti.

Tento příspěvek ukazuje na případové studii pekárny NOPEK z Vysokého Mýta, jak může aplikace aproximačních metod pomoci při plánování rozvozu pečiva zákazníkům. Při optimalizaci jednoho z tzv.

"rychlých rozvozů" se tak podařilo snížit počet vozidel, která byla zapotřebí, o 18%. Podobná úspora u všech "rychlých rozvozů" ve firmě může znamenat pokles ceny majetku (vázanosti kapitálu v tomto majetku) o 17 mil. Kč. Zároveň dojde ke zvýšení zisku o 0,6 mil. Kč a ke zvýšení rentability až o 2,5%. Navíc jsou vozidla rovnoměrněji využita, čímž odběratelům může být dodáno zboží v termínech, které jim lépe vyhovují.

Klíčová slova

Rozvoz pečiva, trasovací problém, aproximační metoda.

Introduction

Rozvoz pečiva, trasovací problém, aproximační metoda. The problem of the delivery optimization of a specific material can in reality be encountered with very often. The delivery is usually realized by a circular or round trip which, in comparison with the realization of each route from the supplier to the consumer, saves expanses for individual gateways from the same supplier and/or trips to one consumer. There exist many tasks of this kind and in general they are referred to as vehicle routing problems (VRP). Most of them belong among the so-called NP-complete or NP-hard problems. This type of tasks is distinguished by a non-existent effective algorithm which would be able to find their precise theoretical optimum. All known approaches capable of this task need the number of operations increasing exponentially with the growth of data (the number of places, suppliers or consumers), which is basically the same amount as when solving the task using "brute force", that is by calculating the values of an objective function for all possible task solutions and selecting the one whose value was found as optimal. Contemporary computer technology enables such task solutions on most effective devices within the scope of 20 places maximum; and in respect to the mentioned exponential dependency, we can assume that this number will even in the future grow only very slowly in spite of rapid computer technology development. Therefore, for these types of tasks there are created the so-called heuristics (approximation methods) offering on the one hand only approximate solutions, on the other hand, however, they are so high-quality that we can regard them as economic optimums.

The most "classical" of these tasks is the traveling salesman problem (TSP). In this case the transportation among all serviced places is to be realized by one circle. It is possibly the most solved type of "round" tasks, whose solution is at the same time a part of the solution of some VRP types. It is also one of the most solved tasks belonging among the NP-complete problems and there exist many types of heuristics for this type of task. However, very often we deal with more complicated situations. Reasons, why one round trip is not enough can vary, e.g. small vehicle capacity, distribution is necessary in due time during which it is impossible to reach all places. Above all, individual suppliers or consumers may have other special demands. In this case it is necessary to create more circles, i.e. more than one vehicle must depart from a central standpoint or, one vehicle must make more round trips and other places or standpoints (suppliers, consumers) must be reasonably divided into groups that will each be serviced during one round trip. There are many VRP types; however, a practical occurrence of individual types is less common than in case of TSP and that is why their choice of heuristics is not as wide as for TSP. The conditions of individual VRP cases are very often so specific that they do not even respond to any of the studied types. Therefore, we usually obtain VRP heuristics by the modification of TSP heuristics.

There exists no generally used software for TSP and VRPs practical solution; the first programs have started to appear on the Internet only recently. In fact, users have no chance to find out which particular method (heuristics) they use. Companies usually solve these tasks "manually" without the use of any specific method even in such cases when in other circumstances, as for instance collecting data for this type of task, they make use of the latest computer technology.

This is also the case of NOPEK Bakery, the focus of our article. The firm quarters are situated in Vysoké Mýto. The main program is the manufacture of bakery, patisserie and gingerbread products using traditional and industrial means of manufacture. Other activities involve trade, including wholesale activities and the maintenance of technical equipment for bakeries and patisseries. NOPEK Bakery has several plants in Hořice, Hrušová, Lanškroun, Svitavy, Vysoké Mýto, Moravská Třebová and Česká Třebová. The company also has its own stores: two are found in Vysoké Mýto and Moravská Třebová, and one is found in Jevíčko, Osík by Litomyšl, Velké Opatovice and Hrušová. Besides this the company runs a non-smoking coffee shop in Vysoké Mýto and a patisserie in Svitavy. The company central storehouse is located in Hrušová. This and other information concerning the company can be found at [20].

Based on annual reports, NOPEK bakery reached 158,5 million CZK in sales and 246,0 million CZK in outputs during 2007 and 177,7 million CZK in sales and 271,7 million CZK in outputs during 2008. The turnover volume was 404,5 million CZK in 2007 and 449.4 million CZK in 2008. In 2008 the company had 356,2 million CZK tenure out of which 241,6 million CZK amounted to buildings and machinery. On the whole, the share of the buildings and machinery on the tenure was 67.84%. The earnings for the fiscal year 2007 came to 21,3 CZK million and for the fiscal year 2008 to 14,7 million CZK, i.e. the tenure profitability was 6.94% in 2007 and 4.11% in 2008. The figures presented refer to the two years 2007 and 2008 because the data used in this article represent 2008.

For dough transportation the company uses Iveco brand trucks with the capacity of 600 crates and Avia trucks with the capacity of 400 crates. The bakery has contracts with both, retailers as well as supermarkets. It is not the vehicle capacity that determines the number of gateways as strict demands on behalf of supermarkets and bigger chains. All in all, we can distinguish three types of delivery: "fast delivery", "long delivery" and "special orders". The situation is illustrated in Figure 1.

"Long delivery" usually takes a longer distance to other regions of the Czech Republic. Suppliers are most likely large businesses, department stores, supermarkets, camps, school events etc., which require a greater amount of goods, making simpler routes usually with four suppliers at the most. These routes can be easily optimized by drivers themselves. Not even special orders give much space for optimization. They are usually placed by the largest companies and big chains whose typical representatives are supermarkets. These wholesalers demand strict meeting deadlines of their orders, and if the deadlines are not met, they may decide to change the supplier. The NOPEK Company may thus lose its clients. Usually the company sends trucks specifically for them. The trucks are usually filled only with approximately 25-30% of goods, primarily from the nearest storehouse or from the central storehouse if the nearest one has no goods available.

"Fast delivery" is defined by the company management as an area which has its own distribution plan. Furthermore, such an area is supplied from one storehouse (while more "fast deliveries" can be performed from this particular storehouse). It contains several tens of places (towns, villages) with usually more than one retailer. They usually show certain stability in their demands, that is they order the same amount of goods basically every day (this of course accounts also for weekend orders of consumers performing also at weekends). The goods must be delivered before their opening hours but, since it is usually easiest for them to take goods from local bakeries, they are always willing to make compromises when it comes to a delivery deadline. In respect to the number of consumers and a limited vehicle capacity it is necessary to use several vehicles for each area. This all opens a large space for optimization regarding not only delivery distribution among individual vehicles, but particular routes for each vehicle.

The aim of this article is to apply different heuristics on one of these "fast deliveries", compare obtained results with real bakery delivery and demonstrate the effectiveness of these methods. Some of the results (as well as the data mentioned in Introduction and other information concerning NOPEK Company) have been adopted from [10] and subsequently have been completed by our own calculations.

Case studies on delivery planning have recently been published quite frequently. In some cases commercial software is used for solution as in, for example, studies from the central Finland which concern a route proposal for seniors home care [1]

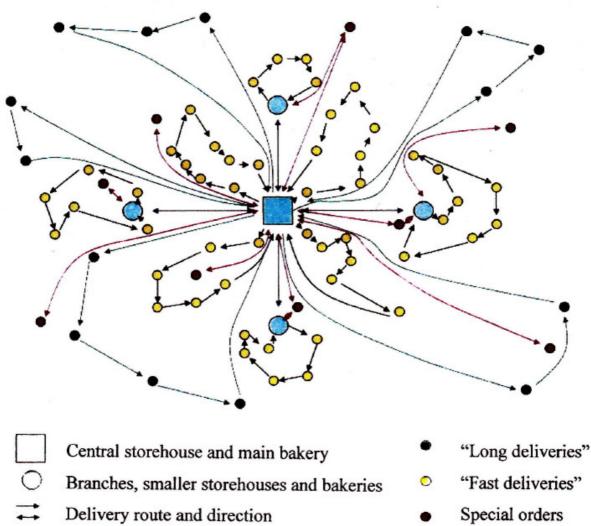


Figure 1.

and especially for food delivery to their homes [2] where significant savings were reached in comparison with the former delivery organization, or in fuel oil distribution for Petramina company from its depot to gas stations in one Jakarta district [13] where the new proposal also brought some savings. In other cases heuristics are used for a solution, as for instance in the optimization of the municipal refuse collecting system where costs were reduced significantly [11], or in planning lumber haulage [16] where one of the visited heuristics which even proved as relatively successful and suitable for this purpose was also the savings method, i.e. one of the methods used in this article. [14] and [15] compares the heuristics application with exact computation using integer programming (which in its time-difficulty corresponds to using "brute force") with the case of the transportation of the University of The Thai Chamber of Commerce employees by university buses to work. While the exact computation brought the result in sensible computational time only in some cases and in others it failed, using the heuristics results were obtained within a small amount of computational time and were good in comparison with the exact computation, provided it was successful.

Material and Methods

The article deals with one "fast delivery" from the central storehouse in Hrušová. Its consumers, all situated in the area with a dense road net in the distance of 50 km from the storehouse, can be divided into three groups based on the time of their need of goods delivery. We will further refer to these groups as time zones. Time zone 1 includes small village shops, local bakery branch stores, large businesses and cooperative farms. They demand the delivery till 5:00 or 5:30 a.m. at the latest and they will certainly not take goods before 1:00 a.m. Time zone 2 includes shops which normally open around 8:00, that is discount stores

and supermarket in Vysoké Mýto. Their time deadlines are less flexible, especially their uttermost limit 8:00 a.m. cannot be crossed. Time zone 3 also includes big large businesses, cooperatives and cafeterias preparing mid-day meals. There the goods may be delivered by 11:00 a.m. In respect to vehicle capacity and consumer demands, each vehicle will make the delivery only within one time zone and for the supply of each time zone several vehicles will be needed. The delivery for each of these time zones will be solved as an individual task.

From the above mentioned facts we can assume that the main limitation of delivery planning is a time interval during which it is necessary to carry out the delivery. Just as well, we can use time as an optimization criterion (objective function). The task for an individual time zone is to a great extent similar to the time limited vehicle routing problem (TLVRP) described in [9], where it concerns, strictly speaking, the optimization of delivery from the central standpoint to a certain amount of other standpoints (or in the opposite direction), which should be performed to a certain time limit. However, our delivery transportation plan in question is rather different in several points from the task just mentioned. To the time needed for the vehicle delivery we need to add time which is needed in every standpoint for unloading goods, and if there is more than one business in a standpoint we also need to add time necessary for trans fer from one business to another. Contrastingly, from the viewpoint of consumers, the time of vehicle departure from the storehouse and the time of its arrival back are unimportant. Therefore, only the period from the arrival of the vehicle to the first consumer till its departure from the last consumer will be included to the time limit that should not be crossed. We will refer to this time consumed by individual vehicles as neat delivery time (NDT). Nonetheless, we will also observe gross delivery time (GDT), that is time which the vehicle spends on its way from its departure till its arrival in the storehouse. It is also important to pay attention to the vehicle capacity so that it is not exceeded.

However, for a quantitative task definition it is necessary to define its cost matrix. Costs will be the time required for the transfer between two standpoints available in the information system [19]. In case there is more than one business serviced in one place, there will be extra 5 minutes added the each transfer between two consecutive consumers. Generally, we may express this added time by the formula $(n-1)\cdot5$, where n is the number of consumers in a standpoint. Furthermore, we need to add the time necessary for unloading goods. This will present 5 minutes for each 36 crates. These additional times were calculated on the basis of drivers' practical experience and we shall call them manipulation times. The cost matrix, manipulation times and the number of crates ordered by individual consumers in each standpoint for the time zones 1, 2 and 3, are shown in Table 1, 2 and 3 respectively.

The first heuristics, used during solving the problem, was the nearest neighbor method (NNM). Actually, it is the simplest known method based on the fact that from each standpoint we continue to the nearest so far unvisited standpoint (following a route with the most convenient cost). However, its obvious deficit is the fact that sectors included in the circle as last ones have inconvenient costs. which reduces the overall solution value. Rosenkrantz, Stearns and Lewis [12] tested the functioning of this method for TSP, and their findings confirmed its expected very low quality. Three different modifications were tested in [10] for our problem. The first one (further referred to as version 1) began with the construction of each circle in the central storehouse and continued to the nearest standpoint till the time limit for the NDT or vehicle capacity was exceeded. Afterwards, the vehicle returned to the central storehouse. In fact, this approach shows analogy with the method mentioned for TLVRP in [9]. However, solutions obtained by this approach displayed inconvenient ordering of the most remote standpoints from the central storehouse. Version 2 attempts to eliminate this deficit. The first visited standpoint during each route is the most remote one from the central storehouse out of standpoints so far not included in previously constructed routes, and from there we again move to the nearest one. In case of version 3 it is rather questionable whether the NNM modification is still concerned. The circles are constructed parallel (all at once) so that first the costs are ordered from the most convenient one to the least convenient. Then they are processed in this order so that each particular segment is added to the solution under condition that in case of each route it

	Vysoké Mýto	Sedliště	Roveň	Rosice	Přestavlky	Pardubice	Osík	Němčice	Moravany	Litomyšl	Kostěnice	Chrudim	Chrást	Hrochův Týnec	Honbice	Holice	Hrušová	manipulation time	number of crates
Vysoké Mýto	-	13	21	28	27	40	20	21	25	19	32	37	34	23	29	20	5	95	302
Sedliště	13	-	37	41	40	53	10	11	38	6	45	50	47	36	42	33	8	10	55
Roveň	21	37	I	26	22	25	41	42	9	37	12	31	30	17	24	11	26	20	39
Rosice	28	41	26	-	9	40	49	50	19	44	18	22	6	9	11	34	33	10	14
Přestavlky	27	40	22	9	-	27	48	49	14	44	14	18	10	4	8	30	32	10	15
Pardubice	40	53	25	40	27	-	60	61	25	56	18	17	36	22	28	23	45	35	87
Osík	20	10	41	49	48	60	-	11	46	6	53	57	50	44	50	41	16	15	59
Němčice	21	11	42	50	49	61	11	-	47	6	54	58	56	45	51	44	20	5	58
Moravany	25	38	9	19	14	25	46	47	I	41	8	23	22	1	20	17	30	10	15
Litomyšl	19	6	37	44	44	56	6	6	41	-	49	53	50	39	46	39	11	105	287
Kostěnice	32	45	12	18	14	18	53	54	8	49	-	22	22	9	16	19	37	15	20
Chrudim	37	50	31	22	18	17	57	58	23	53	22	-	21	13	14	38	42	90	240
Chrást	34	47	30	6	10	36	50	56	22	50	22	21	-	13	11	39	39	15	18
Hrochův																			
Týnec	23	36	17	9	4	22	44	45	1	39	9	13	13	-	6	26	28	25	27
Honbice	29	42	24	11	8	28	50	51	20	46	16	14	11	6	-	32	34	5	6
Holice	20	33	11	34	30	23	41	44	17	39	19	38	39	26	32	-	25	10	35
Hrušová	5	8	26	33	32	45	16	20	30	11	37	42	39	28	34	25	-	-	-

Table 1.

[70]

	Vysoké Mýto	Újezdec	Roveň	Nová Sídla	Moravany	Makov	Kostěnice	Javorník	Chacholice	Hrušová	Holice	H. Újezd	Džbánov	D. Újezd	Cerekvice	Bučina	Chotovice	Nové Hrady	Proseč	Pňluka	Suchá Lhota	Leština	Hrušová	manipulation time	number of crates
Vysoké										_			_		_										
Mýto	-	13	21	12	25	18	32	14	36	5	20	30	5	25	6	9	20	22	33	17	13	15	5	75	226
Újezdec	13	-	33	7	37	5	45	13	39	7	32	20	12	14	5	8	8	12	24	10	12	15	7	15	48
Roveň	21	33	-	33	9	38	12	35	34	26	11	51	26	46	27	30	41	42	54	38	34	36	26	10	26
Nová Sídla	12	7	33	-	37	9	44	13	43	7	33	19	13	14	5	8	12	16	28	16	12	20	7	5	4
Moravany	25	37	9	37	-	43	8	40	26	30	17	56	32	51	31	34	46	44	55	42	39	38	30	10	10
Makov	18	5	38	9	43	-	50	17	33	13	38	14	22	11	11	12	2	6	18	9	7	12	13	10	36
Kostěnice	32	45	12	44	8	50	-	47	26	37	19	63	38	58	39	41	50	46	55	50	46	43	37	5	8
Javorník	14	13	35	13	40	17	47	-	32	10	35	32	6	21	9	5	20	10	21	13	9	6	10	5	4
Chacholice	36	39	34	43	26	33	26	32	-	41	43	49	37	45	41	37	30	36	35	31	32	24	41	5	7
Hrušová	5	7	26	7	30	13	37	10	41	-	25	26	5	21	1	4	15	17	28	12	8	16	0	35	128
Holice	20	32	11	33	17	38	19	35	43	25	-	51	28	46	26	29	41	43	53	37	34	36	25	45	140
H. Újezd	30	20	51	19	56	14	63	32	49	26	51	-	31	5	25	27	17	21	18	24	22	27	26	15	62
Džbánov	5	12	26	13	32	22	38	6	37	5	28	31	-	26	6	12	20	16	26	20	16	13	5	5	3
D. Újezd	25	14	46	14	51	11	58	21	45	21	46	5	26	-	20	22	14	18	22	20	19	23	21	30	45
Cerekvice	6	5	27	5	31	11	39	9	41	1	26	25	6	20	-	4	14	16	28	12	8	16	1	20	80
Bučina	9	8	30	8	34	12	41	5	37	4	29	27	12	22	4	-	15	12	24	8	4	12	4	5	8
Chotovice	20	8	41	12	46	2	50	20	30	15	41	17	20	14	14	15	-	4	15	6	10	9	15	10	22
Nové Hrady	22	12	42	16	44	6	46	10	36	17	43	21	16	18	16	12	4	-	11	4	8	6	17	20	45
Proseč	33	24	54	28	55	18	55	21	35	28	53	18	26	22	28	24	15	11	-	16	19	18	28	35	105
Příluka	17	10	38	16	42	9	50	13	31	12	37	24	20	20	12	8	6	4	16	-	3	9	12	10	9
Suchá Lhota	13	12	34	12	39	7	46	9	32	8	34	22	16	19	8	4	10	8	19	3	-	9	8	5	5
Leština	15	15	36	20	38	12	43	6	24	16	36	27	13	23	16	12	9	6	18	9	9	-	16	5	8
Hrušová	5	7	26	7	30	13	37	10	41	0	25	26	5	21	1	4	15	17	28	12	8	16	-	-	-

Table 2.

	Vysoké Mýto	Tržek	Sedliště	Řestoky	Rosice	Přestavlky	Pardubice	Němčice	Nabočany	Litomyšl	Chrudim	Chrást	Chacholice	Hrochův Týnec	Honbice	Holice	Dašice	D. Újezd	Nové Hrady	Proseč	Hrušová	manipulation time	number of crates
Vysoké Mýto	-	11	13	31	28	27	40	21	27	19	37	34	36	23	29	20	30	25	22	33	5	75	226
Tržek	11	-	3	43	39	39	51	12	39	7	48	43	44	34	41	31	42	15	17	28	6	5	9
Sedliště	13	3	-	44	41	40	53	11	40	6	50	47	51	36	42	33	46	18	21	37	8	10	54,5
Řestoky	31	43	44	-	6	3	31	53	7	48	18	6	10	8	5	34	20	50	32	39	36	20	70
Rosice	28	39	41	6	-	9	40	50	11	44	22	6	10	9	11	34	22	47	30	40	33	5	8,8
Přestavlky	27	39	40	3	9	-	27	49	7	44	18	10	13	4	8	30	18	53	36	43	32	5	3,6
Pardubice	40	51	53	31	40	27	-	61	27	56	17	36	40	22	28	23	15	65	61	69	45	80	203
Němčice	21	12	11	53	50	49	61	-	49	6	58	56	57	45	51	44	52	16	28	38	20	5	28
Nabočany	27	39	40	7	11	7	27	49	-	44	12	13	17	4	2	30	18	53	39	46	32	5	6
Litomyšl	19	7	6	48	44	44	56	6	44	-	53	50	50	39	46	39	47	11	23	32	11	75	191
Chrudim	37	48	50	18	22	18	17	58	12	53	-	21	25	13	14	38	22	62	51	55	42	60	160
Chrást	34	43	47	6	6	10	36	56	13	50	21	-	4	13	11	39	26	44	27	34	39	10	12
Chacholice	36	44	51	10	10	13	40	57	17	50	25	4	-	17	15	43	30	45	36	35	41	5	7
Hrochův																							
Týnec	23	34	36	8	9	4	22	45	4	39	13	13	17	-	6	26	13	49	37	46	28	10	11,4
Honbice	29	41	42	5	11	8	28	51	2	46	14	11	15	6	-	32	23	54	37	44	34	5	6
Holice	20	31	33	34	34	30	23	44	30	39	38	39	43	26	32	-	16	46	43	53	25	55	175
Dašice	30	42	46	20	22	18	15	52	18	47	22	26	30	13	23	16	-	56	50	60	35	10	17
D. Újezd	25	15	18	50	47	53	65	16	53	11	62	44	45	49	54	46	56	-	18	22	21	15	30
Nové Hrady	22	17	21	32	30	36	61	28	39	23	51	27	36	37	37	43	50	18	-	11	17	10	30
Proseč	33	28	37	39	40	43	69	38	46	32	55	34	35	46	44	53	60	22	11	-	28	15	45
Hrušová	5	6	8	36	33	32	45	20	32	11	42	39	41	28	34	25	35	21	17	28	-	-	-

Table 3.

does not violate the constraints given by NDT and vehicle capacity. Initially, the solution has contained individual pairs of standpoints which gradually connect into more complex routes. This approach is in analogy with the Borůvka [3] and Kruskal [6] algorithm for a minimum spanning tree in a graph.

As the next method we applied the savings method (SM) by Clarke and Wright [4]. It is based on the use of savings rather than costs, which are calculated as differences between the length of the route across another previously selected standpoint (the same for the calculation of all savings) and the length of a direct route (cost) between two given standpoints. For the VRP applications we use a central standpoint from where all vehicles depart. [10] uses three versions of savings method which function just the same as the above mentioned NNM modifications, making a difference only in the fact that they use savings matrix in place of costs. Version 3 is at the same time analogical to a parallel SM modification for TLVRP from [7]. Unlike [10], we further even tested version 4 which constructed routes for individual vehicles in sequence (one after another) but its starting point was a segment with the lowest savings out of so far non-included ones (it constructed routes "from the middle"). In fact, it concerns the approach analogical to the SM application for TLVRP from [9].

The last method modified in [10] for the tested problem of bakery products delivery was a loss method [17], [18]. Generally, e.g. in the TSP, the quality of the solution obtained by this method is to a great extent positively influenced by the fact that following each step (inserting a certain segment into the solution), we leave out costs of those segments which cannot become parts of the solution and, based on this, we recalculate the losses. However, solving VRPs, we can find all these segments only with difficulty, and also [10] does not do it sufficiently enough. No wonder that method did not present the best solution for either of the time zones. On each occasion of our testing some tested methods proved to be more successful. Therefore we do not mention our obtained findings in this article.

Moreover, we also tested one approach not mentioned in [10] at all. We attempted to apply

Habr frequencies [5] to the optimal delivery calculation. As well as savings, Habr frequencies are determined fore each cost (a direct route between two standpoints) but unlike savings, they have the advantage in the fact that when evaluating them all other costs are taken into account, even those which do not concern the route in question (in cost matrix they are not found in the same line or column). The applied approach was analogical to the method for TLVRP from [8] including the formula taking into account a specific role of a central standpoint from where vehicles depart. It is at the same time similar to NNM and SM versions 3 described above.

Results and Discussion

Table 4 presents all variants of the delivery plans obtained by the individual methods, including the approach formerly used by the bakery. For each time zone, the obtained best solutions are emphasized. Sometimes it is difficult to decide which solution is really the most appropriate, e.g. whether to give a priority to a "mathematically" optimal solution (with the shortest sum of the NDT or/and GDT of all vehicles but with the NDT of individual vehicles at a different length) or a solution where the total is higher; however, there is not a great difference in NDT between the routes of the individual vehicles. In this case more variants have been marked as the good ones.

The contribution of the methods mainly lies in the reduction of the number of employed vehicles. Whereas the firm needed four vehicles for the time zones 1 and 3 and three vehicles more for the time zone 2, that is 11 vehicles in total, almost all solutions obtained by individual methods needed only three vehicles for each time zone (except only one method in one time zone), that is 9 in total. This of course means 18% of savings, with regard to the number of vehicles. Moreover, the methods are capable of finding solution with a better balance of NDT and GDT of the individual vehicles and with the GDT being 10% shorter. On the contrary, we were practically not able to improve the NDT. However, this is the fact that we expected due to a short distance among particular customers.

Further, it is worth noticing that for every time zone (i.e. for every partial task) each method offered a different solution. By applying more methods, the user may obtain more delivery plans and select

			Ti	me zone 1	l			Time z	one 2		Time zone 3							
		vehicle1	vehicle2	vehicle3	vehicle4	in total	vehicle1	vehicle2	vehicle3	in total	vehicle1	vehicle2	vehicle3	vehicle4	in total			
Former	NDT	1,75	2,78	4,32	3,25	12,10	3,60	4,38	3,67	11,65	4,02	3,93	4,43	2,35	14,73			
plan	GDT	1,58	2,57	2,90	2,00	9,05	3,43	3,62	2,25	9,30	3,85	3,03	3,02	1,60	11,50			
NNM v.1	NDT	3,73	4,28	4,22	-	12,23	3,14	3,61	3,25	10,00	5,13	4,10	4,48	-	13,71			
	GDT	3,47	3,63	3,23	-	10,33	3,21	3,24	2,15	8,60	4,15	2,93	3,50	-	10,58			
NNM v.2	NDT	4,75	4,44	4,07	-	13,26	4,67	3,17	3,45	11,29	5,10	4,48	3,95	-	13,53			
ININIVI V.Z	GDT	3,28	3,61	3,72	-	10,61	3,63	2,47	3,28	9,38	4,27	3,50	3,47	-	11,24			
NNM v.3	NDT	4,22	4,06	3,95	-	12,23	3,65	3,37	4,58	11,60	4,22	4,05	5,47	-	13,74			
	GDT	3,15	3,64	3,12	-	9,91	3,30	3,02	3,73	10,05	2,87	3,78	4,25	-	10,90			
SM v.1	NDT	3,87	5,32	5,28	-	14,47	4,08	3,73	2,73	10,54	4,50	4,47	4,20	-	13,17			
SIVI V.1	GDT	3,03	3,98	4,37	-	11,38	3,52	3,53	2,62	9,67	3,72	3,90	3,37	-	10,99			
SM v.2	NDT	4,92	4,23	3,63	-	12,78	4,08	3,87	2,95	10,90	4,95	4,28	4,57	-	13,80			
51VI V.2	GDT	3,70	3,72	3,28	-	10,70	3,73	3,52	2,10	9,35	3,52	3,60	4,02	-	11,14			
SM v.3	NDT	4,60	4,07	3,95	-	12,62	3,98	3,47	2,88	10,33	4,47	4,65	4,37	-	13,49			
SIVI V.3	GDT	3,53	3,43	3,68	-	10,64	3,57	3,00	2,60	9,17	3,42	3,88	3,82	-	11,12			
SM v.4	NDT	5,08	4,63	3,10	-	12,81	4,20	3,52	2,72	10,44	3,92	5,37	4,62	-	13,91			
31VI V.4	GDT	3,82	4,02	2,70	-	10,54	3,32	3,32	2,60	9,24	2,67	4,48	4,43	-	11,58			
Hohn frog	NDT	4,67	4,95	4,43	-	14,05	4,63	3,97	3,57	12,17		colution .	with form	vahialaa				
Habr freq.	GDT	4,35	3,97	3,47	-	11,79	3,77	3,65	3,55	10,97		solution with four vehicles						

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Table 4.

among them the one which s/he finds the most favorable. What is more, every time (in each time zone) a different method succeeded, including NNM or sequential versions of methods from which we could theoretically expect worse results. Therefore, it is worth testing more different methods during calculations.

Are we to express the benefit in monetary units, we can perceive the situation in several ways. The easiest way is to express costs per 1 km ride, find out how many kilometres the company drove according to the original delivery plan and compare it with a number of kilometres driven according to the new plan using the methods. To determine the costs we used the sum which is charged by services providing car transport. In companies using Avia vehicles, e.g. REFIT95 spol. s.r.o. (REFIT95 Ltd.), we can get the transport starting at 15 CZK per km and owing to the fact that this sum differs from Iveco vehicles only minimally, e.g. they are only about 2 CZK per km more expensive as it is the case of Tavočer s.r.o. (Tavocer Ltd.), we will further consider this sum. We have described the situation by the time of the ride and in order to transfer time to distance we need to know an average speed of vehicles during delivery. Let us suppose it is 50 km per hour. Having added all three time zones, the overall HDR is about two hours and a quarter shorter for the new solution and thus it saves approximately 200 CZK per every work day. Annually the savings will amount to 60,000 CZK. This concerns only a single "fast delivery", assuming there are 10 altogether. If we manage such savings during each "fast delivery", the annual costs savings as well as the increase in profit will amount to 0,6 mil. CZK, which means the increase in profitability between 1.6 and 2%.

The second type of benefit numeration is the comparison of the situation in the company before installing the optimization with the situation when the number of vehicles owned by the company was lower by the number of the vehicles saved by the company after applying the proposed optimization of a delivery plan. If the number of vehicles in each of the ten "fast vehicles" is reduced by two as in the case presented in this article, altogether 20 vehicles will be saved. In regard to the fact that one Avia vehicle costs about 850,000 CZK, the tenure of the company would be by 17 million CZK less. Further it is necessary to consider the profit increase by 0,6

mil CZK reached by saving the costs shortening the overall length of delivery routes (enumerated above in the first type of benefit formulation). The increase in profitability amounts to between 2 and 2.5% in this case.

Another optimization benefit which, however, is not possible to quantify, represents costs savings on the basis of better organization of a delivery process. It mainly presents the possibility to deliver bakery products to consumers in times which suit them better and thus improves mutual supplierconsumer partnership.

If we assess the effectiveness of a method application, we must also consider "time availability" or "stability" of the model. Vendors and businesses, supplied with goods from the bakery, reflect changes in demand of their customers and, correspondingly, they render their demands for the bakery. The majority of permanent consumers included in "fast delivery" modifies their demands every month and confirms their order for every following month. Real particular demands may differ a little from the negotiated ones during week days; however, this does not prevent anybody from the realization of stable delivery plans. Applying the methods, the calculation can be therefore made only monthly. Nevertheless, this need not be necessary, at least not for some time zones, because changes in demand may be so insignificant that using the present routes, time or capacity limitations will not be violated. In such case it is still possible to use existing routes of delivery and there is no need for the calculation of new ones. There will only be minor modifications in the times of delivery to individual customers.

Besides these regular customers, "fast deliveries" will also gradually include other customers who place orders irregularly and by a single application. Fortunately, they are only few. They are typical especially for the time zone 2. In our monitored month, which was appointed for the time of delivery and which is the concern of our article, there were only 65 such demands, i.e. three daily on average. Altogether they were from 15 standpoints and in every one of them it concerned one or two clients at the most. The size of these orders did not exceed three crates at any rate, and from the point of view of the capacity constraints, the size was therefore redundant. There was always enough

room in the vehicle for these extra crates and thus the questions of capacity did not need to be raised when preparing the delivery plan. From the time point of view, each such demand meant a detour of ten minutes in average and five minutes manipulation time for the unloading of goods (in case two irregular customers in one standpoint placed orders on the same day, it provided for another 5 minutes of manipulation time needed for the transfer from one customer to another during one delivery). Every short-term demand provided for the extension of the time of delivery about 15 to 20 minutes, 25 minutes at the most, which is tolerable – as regards time reserve of most routes of individual vehicles – and considering the NDT constraint. When planning the delivery, these irregular orders did not require any other special calculations and what is more, they were in favor of the use of routes obtained from the application of the methods introduced in this article.

The most convenient one of the proposed solutions introduced in [10] and in our article was actually acceptable even for the NOPEK Company. The bakery accepted and applied it with success into its business practice.

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